Hypertree Decomposition for Parallel Constraint Solving

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Outline

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   - The entire constraint network is dead-end free if it can achieve directional arc-consistency [1].

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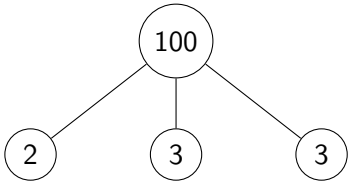
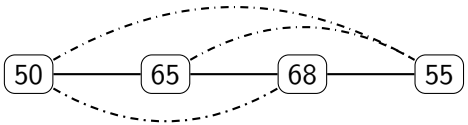


decomposed by det-k-decomp [3]



decomposed by det-k-CP

Introduction

4 How does Det-k-CP work?

- We will answer this question in the following slides.

Structural Decomposition Methods

The existing decomposition methods for constraint network

- Join-Tree-Clustering
- Cycle-cutset Decomposition
- Hinge Decomposition
- Hypertree Decomposition

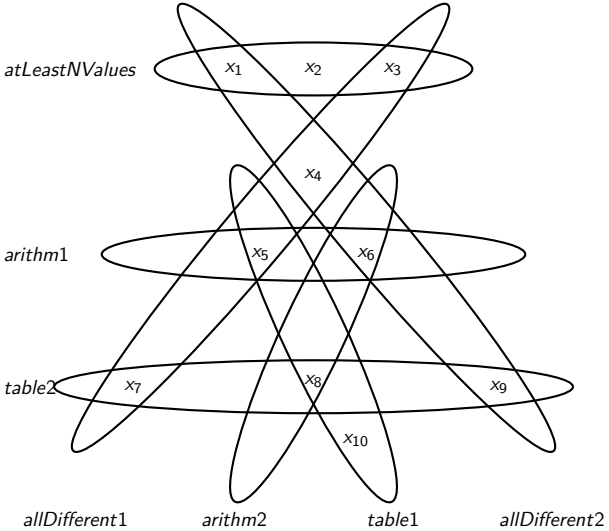
The main objective of Hypertree decomposition is to obtain a decomposition graph as small hypertree width as possible.

Hypertree decomposition example

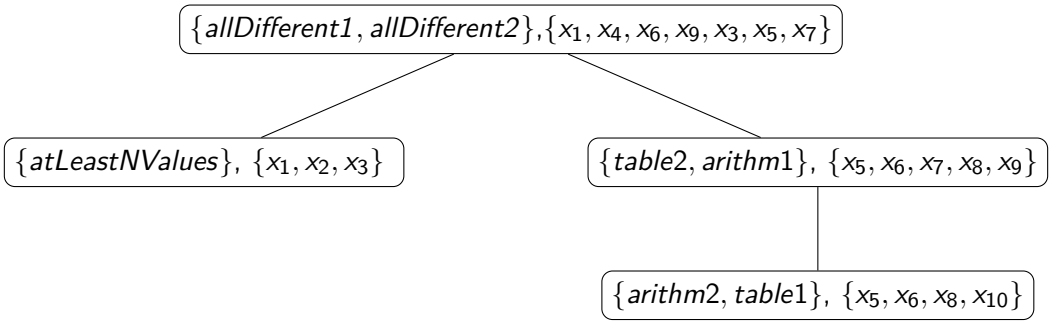
- $allDifferent1(x_3, x_4, x_5, x_7)$
- $allDifferent2(x_1, x_4, x_6, x_9)$
- $atLeastNvalues(x_1, x_2, x_3)$
- $arithm2(x_6, x_8)$
- $table1(x_5, x_8, x_{10})$
- $table2(x_7, x_8, x_9)$
- $arithm1(x_5, x_6)$

The hypergraph for the constraint network.

The example is based on [2]



Hypertree decomposition



One possible hypertree decomposition for the hypergraph on the previous page

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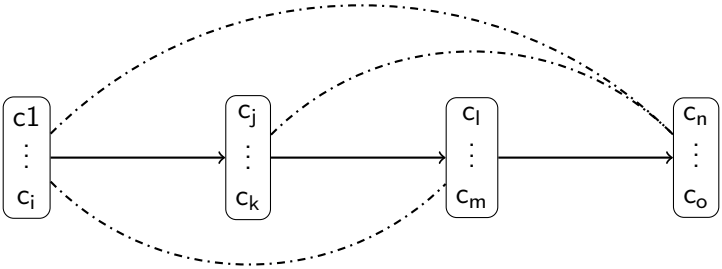

A degenerate decomposition tree for parallel solving.

A target degenerate decomposition tree

An acyclic constraint network can be solved efficiently. [1]

- *Because the constraint problem has a join-tree, its dual problem is a tree of binary constraints and can therefore be solved by the tree-solving algorithm* [1].

An Important Property

One edge between two nodes can be eliminated with-out changing the set of all solutions for the constraint network because there exists an alternative path between the two nodes.

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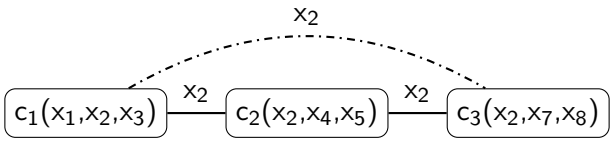
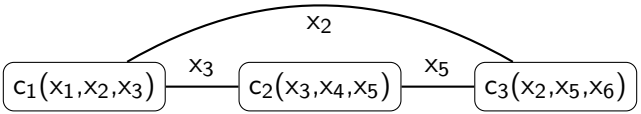


The Principle of det-k-CP



The original simple constraint network

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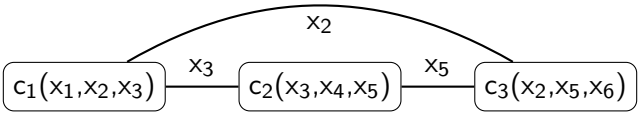

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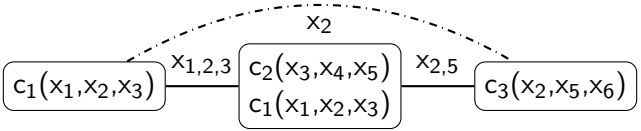
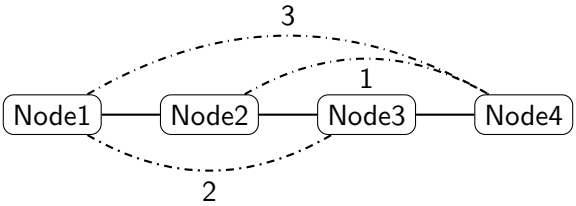


Adding procedure for eliminating non-adjacent edge

The problem caused by simply adding constraint



The arc that had been removed before might occur again due to the subsequent adding procedure.

Outline

Two conditions of Det-k-CP

Mathematically, a decomposed graph after decomposition by *det-k-CP* must meet the following two conditions:

- The shared variables between any pair of nodes (N_p, N_{p+1}) in the interval $[i..j]$ must contain the shared variables between (N_i, N_j), where $i \leq p < p+1 \leq j$.

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- The shared variables between any pair of nodes (N_p, N_{p+1}) in the interval $[i..j]$ must contain the shared variables between (N_i, N_j), where $i \leq p < p+1 \leq j$.
- The decomposition method *det-k-CP* does not lose any constraint or variable.

The main procedure of Det-k-CP

Algorithm 1: The main procedure of Det-k-CP

Input: A Constraint Network N, and the desired number of nodes k.

```
1  Set a list which contains sorted constraints of N based on weight.
2  Evenly distribute constraint onto k nodes.
3  while true do
4  |   get a Potential Solution;
5  |   if The graph passes test conditions (1) and (2) then
6  |   |   break;
7  |   end
8  |   Swap nodes in array_Nodes;
9  end
10 return a degenerated decomposition tree;
```

Get a potential solution for Det-k-CP

Algorithm 2: getPotentialSolution

Output: A potential solution

```
1 Set i_len = the number of nodes;
2 Set i_start = i_len-2;
3 while i_start ≥ 0 do
4     for i_end ← i_start + 2 to i_len do
5         eliminateEdge(i_start, i_end, array_Nodes) ;
6     end
7     Set i_start = i_start − 1 ;
8 end
```

Eliminate an Edge in Det-k-CP

Algorithm 3: eliminate an Edge between node

Input: *i_start*, *i_end*, *array_Nodes*

```
1  for i ← i_start to i_end − 1 do
2      Set list_2BeEliminated = getSharedVariables(i_start, i_end, array_Nodes);
3      Set j = i + 1 ;
4      Set list_SharedOnMainPath = getSharedVariables(i, j, array_Nodes);
5      if list_SharedOnMainPath.notContainsAll(list_2beEliminate) then
6          list_2BeEliminated.removeAll(list_SharedOnMainPath);
7          Set list_2beAddedConstraints = getMinimumSetConstraints4Shared(i, array_Nodes,
             list_2BeEliminated);
8          foreach constraint cs ∈ list_2beAddedConstraints do
9              if array_Nodes[j] notContained cs then
10                 add cs into array_Nodes[j];
11             end
12         end
13     end
14 end
```

Time complexity of Det-k-CP

The asymptotic time of Det-k-CP is $\mathcal{O}(k^4 \cdot N_c \cdot (N_c + N_v))$, where k is the number of nodes in the desired degenerated decomposition tree, N_c and N_v are the number of constraints and the number of variables for the given constraint network respectively.

Outline

Experiment setup

- The benchmark suit for testing det-k-CP is extracted from practical industrial constraint problem [3].
- The algorithms are implemented in Java.
- All the experiments are set up on an iMac computer having an Intel i7-3770 CPU, 3.40GHz, with 8 GB 1600 MHz DDR3.

Results and Conclusion of the experiment

- For the application of parallel constraint solving, the algorithm can be applied to medium scale constraints network with the number of constraints ranging from around 400 to 700.

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- For the application of parallel constraint solving, the algorithm can be applied to medium scale constraints network with the number of constraints ranging from around 400 to 700.
- Competitive execution time as long as the hypergraphs are sufficiently simple. e.g., the constraint network *s5378* with 2958 constraints and 2993 variables) and a big k (e.g., k = 16) takes around two minutes.

Conclusion and Future Work

- The possible ways for improving det-k-CP
  - Take into consideration an estimate of the amount/complexity of computation for the constraints
  - Local search methods might be employed to replace the existing stochastic strategies in order to obtain more optimized decomposition result.
  - Add constraints to one node from other nodes covering the same variables.

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  - Add constraints to one node from other nodes covering the same variables.
- Future Work
  - To develop a related join selection algorithm for det-k-CP to gain the speed up.

Thank you for your attention!
Question?

References

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