THE PROPORTIONAL CONSTRAINT
IN FINITE-DOMAIN CONSTRAINT PROGRAMMING

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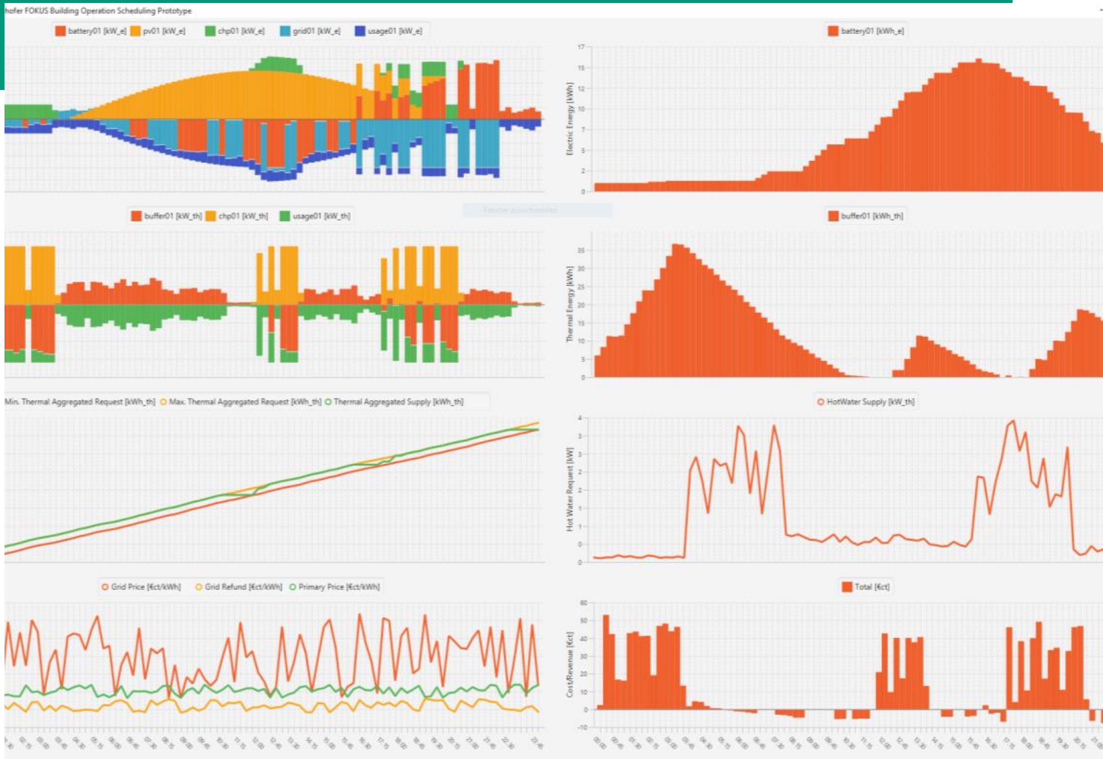
Engineering a Connected World

OPTIMAL CONFIGURATION & OPERATION OF ENERGY SYSTEMS – THE CONTEXT

Sample Configuration

- PV system
- CHP with peak load boiler
- Battery system
- Heat Storage
- Grid connection with variable prices and refunds
- Power requests
  - heating & domestic hot water
  - electricity



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THE PROPORTIONAL CONSTRAINT

Motivation

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Modeling of proportional relations in the context of energy management

Energy loss of storages:

$$W_{t+1} = c \cdot W_t + \cdots , \qquad 0 < c < 1$$

Ratio of thermal and electric power in **C**ombined **H**eat and **P**ower systems:

$$P_{el} = \sigma \cdot \dot{Q}, \qquad 0 < \sigma < 1$$

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The Proportional Constraint – a simple binary constraint:

$t \cdot A = B$, where A, B are Variables and $t \in \mathbb{R}$ is a scalar value

… straight-forward modelling in Linear Programming
… but what about finite integer domain Constraint Programming?

What is an adequate solution of $W = 0.997 \cdot 45689$, where W is integral?

→ $W = 45552 = round(0.997 \cdot 45689)$ seems to be adequate!

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Definition

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$$round(t \cdot A) = B$$

where

- A, B are finite integer domain constraint variables
- $t > 0$ is any real value
- $round(.)$ is the rounding function mapping real values to integer values:
  - $round(x) = \lfloor x + 0.5 \rfloor$, where $\lfloor y \rfloor$ is the greatest integer value less than or equal to y for any real value y.

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Pruning Rules

$$dom^*(B) = dom(B) \cap [round(t \cdot \min(A)), round(t \cdot \max(A))] \qquad (1)$$
$$dom^*(A) = dom(A) \cap [\lceil(\min^*(B) - 0.5) / t\rceil, \langle(\max^*(B) + 0.5) / t\rangle] \qquad (2)$$

where

- $dom(A), dom(B)$ are the finite integer domains of A, B
- $\min(A) = \min\big(dom(A)\big), \max(A) = \max\big(dom(A)\big)$, etc. – for convenience
- $\lceil y \rceil$ is the smallest integer value greater than or equal to y for any real value y

$$\langle x \rangle = \begin{cases} x - 1, & \text{if } x = \lfloor x \rfloor \\ \lfloor x \rfloor, & \text{otherwise} \end{cases}$$

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Correctness of the pruning rules

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… they are *correct*: not any solution is lost → formal proof in the paper

Example:

- Let $round(2.1 \cdot A) = B$ be given where
  - $dom(A) = \{0,1,2,3\}$ and $dom(B) = \{2\}$
- Applying the pruning rules yield

$$dom^*(B) = \{2\} \cap [0,6] = \{2\}$$
$$dom^*(A) = \{0,1,2,3\} \cap [1,1] = \{1\}$$

- Any further iteration will not change the domains
- $\{A \rightarrow 1, B \rightarrow 2\}$ is the solution of this Proportional Constraint

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Termination of iteration and Fixed-Point of the pruning rules

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… pruning always result in a fixed-point after a finite number of iteration due to the fact that

- the domains of the constraint variables are finite
- pruning will not increase these domains (leaving or reducing)

… however, the number of iterations to reach the fixed-point depends on the constraint instance:

Example:

- Let $round(3.0 \cdot A) = B$ be given where
  - $dom(A) = \{1, 2, 3, \dots, n\}$ and $dom(B) = \{3, 5, 8, \dots, 3n - 1\}$
  - $dom^*(B) = dom(B)$ and $dom^*(A) = \{1,2,3, \dots, n-1\}$, etc. → n-1 iterations

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Pruning results in the strongest kind of Bounds Consistency

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Definition [the strongest kind of bounds-consistency]

Let a constraint c with finite integer domain variables $x_1, \ldots, x_n$ be given.

The domains of these variables are $bounds(D)$ consistent, if

for each variable x_i with $1 \leq i \leq n$ and for each $d_i \in \{\min(x_i), \max(x_i)\}$

there exist integers d_j with $d_j \in dom(x_j)$ where $1 \leq j \leq n,\ j \neq i$ such that

*$\{x_1 \rightarrow d_1, \ldots, x_n \rightarrow d_n\}$ is **an integer solution** of c.*

Fixed-point iteration of the pruning rules of the Proportional Constraint always results $bounds(D)$ consistent domains → formal proof in the paper

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Alternative Modelling using Weighted Sum Constraints

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Let $t = p/q$ where p and q are positive integer values, i.e. ***t* be *rational***.
Then for any two finite domain constraint variables A, B the constraint

$$-\frac{q}{2} < q \cdot B - p \cdot A \leq \frac{q}{2}$$

is equivalent to the Proportional Constraint $round(t \cdot A) = B$ in the sense that any solution of one constraint is a solution of the other constraint.

→ formal proof in the paper

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Alternative Modelling – Problem Instances for Run-Time Comparison

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The considered problem instances

- $A(n)$: $t = 0.3$, $dom(A) = \{10, 20, 30, \dots, 10n\}$, $dom(B) = \{3, 5, 8, \dots, 3n - 1\}$.
- $B(n)$: $t = 0.997$, $dom(A) = \{1, 2, 3, \dots, n\}$, $dom(B) = \{1, 2, 3, \dots, n\}$.
- $C(n)$: $t = 0.003$, $dom(A) = \{1, 2, 3, \dots, n\}$, $dom(B) = \{1, 2, 3, \dots, n\}$.

… for $n = 10000, 20000, 40000, 80000$.

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Alternative Modelling – Results of the Run-Time Comparison

Instance	PC avg.	PC best	ALT avg.	ALT best	ALT/PC avg.	ALT/PC best
A(10000)	310.2	297	539.5	440	174 %	148 %
A(20000)	1238.8	1199	1277.8	1227	**103 %**	**102 %**
A(40000)	4035.3	3999	4985.4	4864	124 %	122 %
A(80000)	15582.6	15280	19188.7	18885	123 %	124 %
B(10000)	103.3	78	170.7	109	165 %	140 %
B(20000)	149.5	125	200.1	172	134 %	138 %
B(40000)	196.3	187	278.3	250	142 %	137 %
B(80000)	276.5	250	452.9	406	164 %	162 %
C(10000)	86.0	78	153.6	140	**179 %**	**179 %**
C(20000)	123.4	109	218.8	156	177 %	143 %
C(40000)	191.4	172	281.5	265	147 %	154 %
C(80000)	273.4	250	409.7	359	150 %	144 %

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Generalization

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The Proportional Constraint $round(t \cdot A) = B$ for any $t \in \mathbb{R}$:

– $\boldsymbol{t = 0}$:
  - If $0 \notin dom(B)$ holds, then $dom^*(B) = \phi$ and $dom^*(A) = \phi$
  - If $0 \in dom(B)$ holds, then $dom^*(B) = \{0\}$ and $dom^*(A) = dom(A)$

– $\mathbf{t < 0}$: the pruning rules are adapted accordingly – considering
  - $B = round(-t \cdot -A)$ while pruning the domain of A
  - $-B = round(-t \cdot A)$ while pruning the domain of B

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Conclusion

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- Introduction of a Proportional Constraint in finite integer domain CP
- Definition of according pruning rules
  - Correctness is proven
  - $Bonds(D)$ Consistency is proven
- Alternative modelling is introduced and compared
  - Implementation in firstCS – our own Fraunhofer CP solver
  - Original pruning faster on selected problem instances
- Generalization of the Proportional Constraint for any factor
- Application: optimal operation of energy systems within buildings

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CONTACT & FURTHER INFORMATION

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